



# The Mathematics of Customer Loyalty

## ***Long-term purchase history holds the key***

**T**here is, in general, a high degree of correlation within a purchase data set. Long-time accounts tend to make more purchases, spend more, buy a wider selection of products and place orders more frequently. So, the traditional loyalty indices — total amount, number of purchases, number of products, retention (duration of customer relationship), or recency (time since last purchase) — will tend to identify the best and worst customers. However, these loyalty indices don't provide a consistent evaluation of the middle 75-80% of a firm's customers. A tool with a broader scope is needed to evaluate the average customers — those who are neither the best nor the worst, but form the great middle-majority. Such a tool can recognize that the customer with a consistent record of small purchases may be more loyal than one who made a single large purchase last quarter, and, hence have greater value to the firm.

A good loyalty measurement tool will inevitably be faced with an inconsistent purchase record and must be capable of a meaningful trade-off that recognizes the important relationships within a firm's customer purchase records. Of course, "trade-off" implies that even the best loyalty measurement tool isn't perfect and won't provide as precise an evaluation on a single traditional dimension as measuring that dimension alone. Nevertheless, a loyalty measurement tool should identify distinct populations in which mean or median values conform to the expected relationships on all, or most, of the traditional indices. That is, the "more loyal" customers should, in the aggregate, spend more, purchase more frequently, buy a wider variety of products, etc.

### **CHARACTERISTICS OF A LOYALTY MEASUREMENT TOOL**

As suggested above, there are many alternative measurement indices that can segment a customer population into groups related to what we can call "customer loyalty." Given this situation, what characteristics of a customer segmentation might we use to evaluate or, hopefully, to select the "best" of the available measurements?

**Traditional loyalty indices — total amount, number of purchases, number of products, retention (duration of customer relationship), or recency (time since last purchase) — will tend to identify the best and worst customers. But they will not provide a consistent evaluation of the middle 75-80%.**

If we assign customers to loyalty groups based on their ranking by a "loyalty index," then we expect that:

1. The groups agree with the traditional measures in the statistical sense or, individually, in the "all else equal" sense. Statistically, those groups ranking higher on the loyalty scale should, in the aggregate, have means and medians that agree with the usual concept of customer loyalty. That is, customers in these higher groups have made more purchases, spent more money, purchased a wider variety of products, etc. Furthermore,

when comparing individual customers, if two customers differ only in a single traditional measure, all other measures being equal, the loyalty ranking should conform to the ranking of the traditional measure that differentiates the two.

2. Not only should the means and medians of the loyalty grouping agree with the traditional measurements' concept of "more loyal" but, also, the differences among the groups should be meaningful. This implies that the groups defined by a loyalty index should be statistically different in terms of the multivariate suite of traditional measures.
3. The loyalty index should provide maximal or near-maximal differentiation over purchase amount. The total amount purchased by a customer is, ultimately,

Regardless of a customer's previous loyalty ranking, an atypical prolonged period of account inactivity should be recognized as a possible decline in loyalty that suggests the need for remedial action.

of great importance to the firm and the loyalty grouping should provide as detailed an assessment of this factor as possible within the other constraints described here.

4. The loyalty index should be comparable over time. It is important that the loyalty segmentation provide a base for the assessment of future performance — both at the level of the individual customer and the aggregated customer base. This implies that a second loyalty analysis cannot focus only on the customer base as it exists at the time of the analysis but must be based on the parameters and analysis of the original investigation.
5. The loyalty index should decline due to inactivity. Regardless of a customer's previous loyalty ranking, an atypical prolonged period of account inactivity

should be recognized as a possible decline in loyalty that suggests the need for remedial action.

6. The loyalty index should predict the probability of future account activity. Customer loyalty reflects a propensity or a strengthened relationship with the firm and future purchases. A successful loyalty index will be highly correlated with the likelihood of future purchases.

#### WHY LOYALTY MEASURES BASED ON WEIGHTED SUMS DON'T WORK

It is natural to try to base a loyalty analysis on a weighted sum of the traditional measures. However, there is a major problem with this approach — the need to normalize and force equivalence between the traditional measures.

This need arises first because the measures have widely varying scales. That is, amounts might range to many thousands of dollars, while number of purchases might be less than 10. Second, these measures often exhibit distributions that are severely skewed to the right with many small values and relatively few very large values. That is, the distributions have long tails to the right with the mean much larger than the median. These factors suggest the need for transformation and normalization of the raw attributes to give them each an approximate bell-shaped histogram and a common scale, so they can be reasonably combined without one dominating the others due to scale alone.

To construct a loyalty measure as a sum of transformed and scaled raw measures, it is necessary to determine the weighting factors which are used to convert (transformed and normalized) dollar amounts, number of purchases, variety of purchases, days of retention and days of recency to loyalty units. These weighting factors can, for example, be:

- Determined empirically by using mathematics — a principle-components analysis (PCA) to find the linear combinations of the transformed variables that best describe the variation in the data.
- Selected by an analyst to express the analyst's assessment of the relative importance of the different factors.

The high degree of correlation between the traditional measures will generally lead to agreement on the best

---

and worst of the customer population, regardless of the method applied to assign the weights to the loyalty construct. However, it is the trade-offs required for the majority of customers that differentiates a loyalty index. No matter how the coefficients are chosen, the weighted sum loyalty indices imply a constant trade-off between (transformed) measurement dimensions. That is, “loyalty equivalents” are defined among the different dimensions.

For example, so much “amount” is equivalent in loyalty to so much “number of purchases” regardless of the customer’s “days of retention” or “days of recency.” Or, so many

“days of retention” are loyalty equivalent to so many “days of recency” and so many “number of purchases” regardless of the variety of those purchases, the time between those purchases, or the amount spent on them by the customer.

Loyalty Builders does not feel that the complex nature of “customer loyalty” is amenable to this type of forced equivalence, no matter how the sum components based on the separate measurement dimensions are defined. Rather, the Loyalty Builders’ loyalty index is based on a holistic perspective, provided by a mathematical model that recognizes the natural relationships among the

measurement dimensions to provide non-linear trade-offs dependent, to varying degree, upon the entire customer history, while satisfying each of requirements 1-6 above. ■

*To find out more about what Loyalty Builders LLC can do for your business, go to [loyaltybuilders.com](http://loyaltybuilders.com), or e-mail:*

■ *Mark Klein, CEO*  
*[markk@loyaltybuilders.com](mailto:markk@loyaltybuilders.com)*